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Phase and amplitude control of the inversionless gain in a microwave-driven Λ-type atomic system

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Abstract. In order to achieve the phase-sensitive probe gain without population inversion, we investigate a three-level Λ-type atomic system driven by a coherent field and a microwave field. It is shown that, by modulating the relative phase of applied fields, we can obtain quite high inversionless gain at different probe detunings and change the gain behavior of the probe correspondingly. We find that amplitudes of the coherent field and the microwave field are also important factors that can result in different gain behavior of the probe. Here, we use the microwave field to induce the quantum coherence between the two ground levels, which is necessary for phase-sensitive effects, since it can result in the interference between two different transition channels.

PACS. 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption – 42.50.Hz Strong-field excitation of optical transitions in quantum systems; multiphoton processes; dynamic Stark shift

1 Introduction

In the past decade, much attention in quantum optics has been given to quantum coherence or interference between two neighboring spontaneous decay channels [1–15]. It is found that, when atoms decay spontaneously from an excited level to two near-degenerate ground levels or from two near-degenerate excited levels to a common ground level, the coherence or interference between the two neighboring spontaneous decay channels can lead to many interesting phenomena, for example, the phase control of spontaneous emission [3–6], the modification of absorption and dispersion properties [7–10], and the dynamically induced population inversion [13,14]. This kind of coherence or interference arises from the vacuum of the electromagnetic field, so in the following we call it the vacuum-induced coherence (VIC) for convenience.

Recently, we studied a three-level Λ-type atomic system with two near-degenerate ground levels, and found that we can realize phase control of the probe gain without population inversion due to the presence of VIC [15]. However, the two involved dipole moments $\mathbf{d}_{31}$ and $\mathbf{d}_{32}$ have to be nonorthogonal; otherwise VIC becomes zero for it is proportional to the coupling coefficient

$$\gamma_{12} = \sqrt{\gamma_{31}\gamma_{32}}\frac{\mathbf{d}_{31}\cdot\mathbf{d}_{32}}{d_{31}d_{32}}$$

between levels $|1\rangle$ and $|2\rangle$. Moreover, the frequency difference ($\Delta\omega$) between levels $|1\rangle$ and $|2\rangle$ has to be small as compared to spontaneous decay rates; otherwise VIC disappears due to fast oscillating terms proportional to $e^{\pm i\Delta\omega t}$. We also have to use an incoherent pumping to prepare a small quantity of atoms in the top level $|3\rangle$.

In the present contribution, noting that a microwave field also can generate quantum coherence on a dipole-forbidden transition with two closely lying levels [16–19], we propose a new scheme for three-level Λ-type atoms to achieve the phase-sensitive inversionless gain, where a microwave field and a coherent field are used. In this scheme, the two involved dipole moments can be either parallel or orthogonal, and the frequency difference ($\Delta\omega$) between the two ground levels can be much larger than spontaneous decay rates. Moreover, we can get much stronger coherence between the two ground levels by increasing the strength of the microwave field, which is impossible for VIC.

By numerical simulations, we find that, even if in the absence of an incoherent pumping, we can achieve the phase-sensitive inversionless gain in this microwave-driven atomic system, for the microwave field also can transfer atoms between the two ground levels except induce quantum coherence. When the Rabi frequency of the microwave field is equal to spontaneous decay rates, the gain behavior of the probe is similar to that obtained in reference [15], since the strength of microwave-induced coherence is equivalent to that of VIC. But when the microwave field becomes much stronger, the gain behavior of the probe changes remarkably. That is to say, we also can control the gain property of the probe by modulating the amplitude of the microwave field, *i.e.*, by modulating

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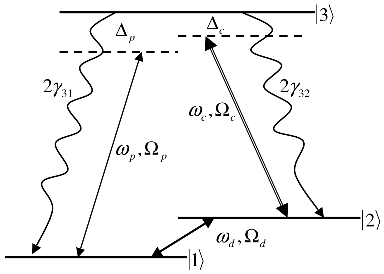

Fig. 1. Schematic representation of a three-level Λ-type atomic system driven by a microwave field, a strong coherent field, and a weak probe.

the strength of the microwave-induced coherence. We also show that, as long as Rabi frequencies of the coherent field and the microwave field are not smaller than spontaneous decay rates, we always can achieve the probe gain on resonance, but we have to tune the relative phase into different regions for different strength of fields.

2 The atomic model and theory

We consider a closed system of three-level atoms in the Λ configuration (see Fig. 1), where transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are assumed dipole allowed but transition $|1\rangle \leftrightarrow |2\rangle$ is assumed dipole forbidden, *i.e.*, levels $|1\rangle$ and $|2\rangle$ have the same parity. A coupling field of frequency (Rabi frequency) ω_c (Ω_c) drives transition $|2\rangle \leftrightarrow |3\rangle$. A resonant microwave field of frequency (Rabi frequency) ω_d (Ω_d) couples level $|1\rangle$ into level $|2\rangle$ through an allowed magnetic transition (*i.e.*, $\omega_d = \omega_{21}$). A weak field of frequency (Rabi frequency) ω_p (Ω_p) is used to probe the gain or absorption on transition $|1\rangle \leftrightarrow |3\rangle$. $2\gamma_{31}$ and $2\gamma_{32}$ are the spontaneous decay rates from level $|3\rangle$ to level $|1\rangle$ and level $|2\rangle$, respectively. $\Delta_p = \omega_{31} - \omega_p$ and $\Delta_c = \omega_{32} - \omega_c$ indicate atom-field detunings of the probe and the coupling field, respectively. In this paper, we assume that ground levels $|1\rangle$ and $|2\rangle$ are closely spaced, but the corresponding frequency difference ω_{21} is much larger than γ_{31} and γ_{32}, so no vacuum-induced coherence terms (proportional to $\sqrt{\gamma_{31}\gamma_{32}}$) exists in the following density matrix equations. With the electric-dipole approximation and the rotating-wave approximation, the density-matrix equations of motion in the interaction picture can be written as:

$$\dot{\rho}_{11} = 2\gamma_{31}\rho_{33} + i\Omega_p^*\rho_{31} - i\Omega_p\rho_{13} + i\Omega_d^*\rho_{21} - i\Omega_d\rho_{12} \tag{1}$$

$$\dot{\rho}_{22} = 2\gamma_{32}\rho_{33} + i\Omega_c^*\rho_{32} - i\Omega_c\rho_{23} + i\Omega_d\rho_{12} - i\Omega_d^*\rho_{21} \tag{2}$$

$$\dot{\rho}_{12} = i(\Delta_p - \Delta_c)\rho_{12} + i\Omega_p^*\rho_{32} - i\Omega_c\rho_{13} + i\Omega_d^*(\rho_{22} - \rho_{11}) \tag{3}$$

$$\dot{\rho}_{13} = (i\Delta_p - \gamma_{31} - \gamma_{32})\rho_{13} + i\Omega_p^*(\rho_{33} - \rho_{11}) - i\Omega_c^*\rho_{12} + i\Omega_d^*\rho_{23} \tag{4}$$

$$\dot{\rho}_{23} = (i\Delta_c - \gamma_{31} - \gamma_{32})\rho_{23} + i\Omega_c^*(\rho_{33} - \rho_{22}) - i\Omega_p^*\rho_{21} + i\Omega_d\rho_{13} \tag{5}$$

$$\rho_{ij} = \rho_{ji}^*. \tag{6}$$

The closure of this atomic system requires that $\rho_{11}+\rho_{22}+\rho_{33} = 1$. To show this atomic system is sensitive to phases of the applied fields, we take the canonical transformation of equations (1–6) as follows: first, considering the initial phases ϕ_p, ϕ_c, and ϕ_d of the probe, the coupling field, and the microwave field, we rewrite Rabi frequencies as $\Omega_p = G_p \exp(i\phi_p)$, $\Omega_c = G_c \exp(i\phi_c)$, and $\Omega_d = G_d \exp(i\phi_d)$, where G_p, G_c, and G_d are chosen to be real; then we redefine atomic variables as $\sigma_{ii} = \rho_{ii}$, $\sigma_{13} = \rho_{13}\exp(i\phi_p)$, $\sigma_{23} = \rho_{23}\exp(i\phi_c)$, and $\sigma_{12} = \rho_{12}\exp(i\phi_p - i\phi_c)$; finally we obtain the equations of motion for the redefined density matrix elements σ_{ij} from equations (1–6) as follows:

$$\dot{\sigma}_{11} = 2\gamma_{31}\sigma_{33} + iG_p(\sigma_{31} - \sigma_{13}) + iG_d(e^{-i\Phi}\sigma_{21} - e^{i\Phi}\sigma_{12}) \tag{7}$$

$$\dot{\sigma}_{22} = 2\gamma_{32}\sigma_{33} + iG_c(\sigma_{32} - \sigma_{23}) + iG_d(e^{i\Phi}\sigma_{12} - e^{-i\Phi}\sigma_{21}) \tag{8}$$

$$\dot{\sigma}_{12} = i(\Delta_p - \Delta_c)\sigma_{12} + iG_p\sigma_{32} - iG_c\sigma_{13} + iG_d e^{-i\Phi}(\sigma_{22} - \sigma_{11}) \tag{9}$$

$$\dot{\sigma}_{13} = (i\Delta_p - \gamma_{31} - \gamma_{32})\sigma_{13} + iG_p(\sigma_{33} - \sigma_{11}) - iG_c\sigma_{12} + iG_d e^{-i\Phi}\sigma_{23} \tag{10}$$

$$\dot{\sigma}_{23} = (i\Delta_c - \gamma_{31} - \gamma_{32})\sigma_{23} + iG_c(\sigma_{33} - \sigma_{22}) - iG_p\sigma_{21} + iG_d e^{i\Phi}\sigma_{13} \tag{11}$$

$$\sigma_{ij} = \sigma_{ji}^* \tag{12}$$

where $\Phi = \phi_d + \phi_c - \phi_p$ is the relative phase of the three fields. From equations (7–12), it is obvious that, due to the existence of the microwave field, this atomic system becomes sensitive to the relative phase Φ, for we cannot eliminate it after the canonical transformation. That is to say, we can change the absorption property, the dispersion property, as well as population distributions of this atomic system just by tuning one of the three phases, for example ϕ_d, and keeping the other two constant. The leading role of the microwave field is to induce the quantum coherence between levels $|1\rangle$ and $|2\rangle$, which is necessary for phase-dependent effects in this atomic system. Moreover, it also can transfer atoms between levels $|1\rangle$ and $|2\rangle$, so we can achieve the probe gain even in the absence of an incoherent field.

As we know, the gain-absorption coefficient on transition $|1\rangle \leftrightarrow |3\rangle$ is proportional to the imaginary part of σ_{13}, and the probe gain will be obtained if $\text{Im}(\sigma_{13}) > 0$. The steady-state solution of $\text{Im}(\sigma_{13})$ is so complicated since it depends on eight parameters: G_p, G_c, G_d, Δ_p, Δ_c, γ_{31}, γ_{32}, and Φ, that we have to resort to numerical calculation in the next section. In what follows, we assume that G_p, G_c, G_d, Δ_p, Δ_c, and γ_{32} are in unit of γ_{31}.

3 Numerical results and qualitative discussion

In this section, in the limit of a weak probe ($G_p \ll \gamma_{31}, \gamma_{32}$), we first investigate effects of various dynamical variables, including G_c, G_d, Δ_p, and Φ, on the probe

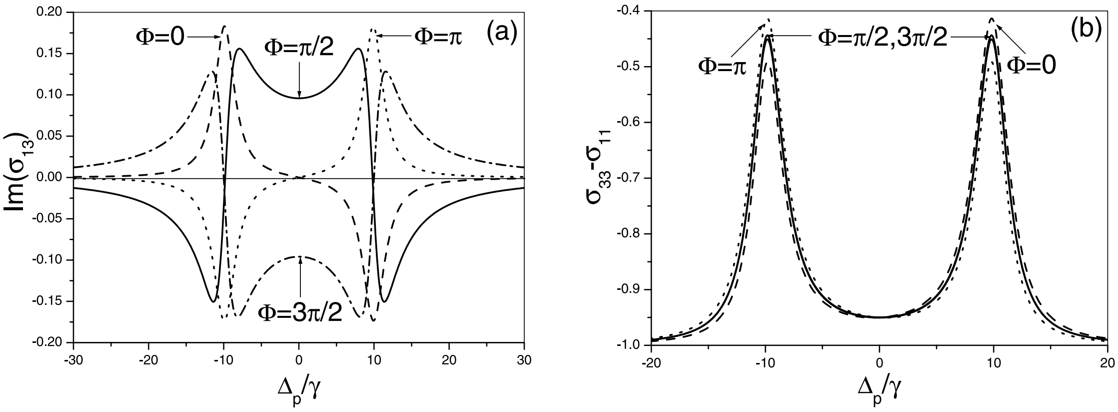

Fig. 2. The probe gain-absorption $\mathrm{Im}(\sigma_{13})$ (a) and population difference $\sigma_{33}-\sigma_{11}$ (b) against the probe detuning Δ_p/γ for different values of the relative phase Φ. Other parameters are $\gamma_{31}=\gamma_{32}=\gamma$, $\Delta_c=0$, $G_p=0.1\gamma$, $G_d=\gamma$, and $G_c=10\gamma$.

gain (or absorption) $\mathrm{Im}(\sigma_{13})$, and then qualitatively interpret the origin of the phase dependence of the probe gain. In the following, just for simplicity, we always choose $\gamma_{31}=\gamma_{32}=\gamma$.

First, we consider the case of $G_d=\gamma$ and $G_c=10\gamma$. In this case, the quantum coherence between level $|1\rangle$ and level $|2\rangle$ generated by the microwave field is in the order of the vacuum-induced coherence, which will have to be considered if level $|1\rangle$ and level $|2\rangle$ lie closely enough. In Figure 2, we plot the probe gain (or absorption) $\mathrm{Im}(\sigma_{13})$ and population difference $\sigma_{33}-\sigma_{11}$ *versus* the probe detuning Δ_p/γ for different values of Φ. It is found that, for the chosen values of Φ in Figure 2, the probe gain without population inversion can be achieved at different probe detunings with different amplitudes. The fact that no population inversion occurs can be seen from Figure 2b. The absence of population inversion on transition $|1\rangle \leftrightarrow |3\rangle$ also holds for other figures where the probe gain is exhibited. Note that, in order to obtain the phase-sensitive probe gain without inversion by the vacuum-induced coherence, we have to use incoherent pumping to produce the necessary population distribution in the top level [15]. However, it is not necessary for the atomic system considered here, because the microwave field can transfer atoms from level $|1\rangle$ to level $|2\rangle$, and then to level $|3\rangle$ with the help of the coupling field.

Now, we consider a special case where the quantum coherence between levels $|1\rangle$ and $|2\rangle$ generated by the microwave field becomes much more stronger. In Figure 3, with $G_d=5\gamma$, we plot the probe gain $\mathrm{Im}(\sigma_{13})$ *versus* the probe detuning Δ_p/γ for different values of G_c and Φ. We find that the gain behavior of the probe in Figure 3 is quite different from that in Figure 2. Comparing Figure 3a with Figure 3c, we can see that, the probe gain $\mathrm{Im}(\sigma_{13})$ with $\Phi=0$ and that with $\Phi=\pi$ are exactly symmetric about $\Delta_p=0$. For $\Phi=0$ ($\Phi=\pi$), the probe gain without inversion only can be obtained in the region of $\Delta_p<0$ ($\Delta_p>0$), and it increases with the increasing of G_c. From Figures 3b and 3d, we can see that, the probe gain with $\Phi=\pi/2$ and that with $\Phi=3\pi/2$ are exactly symmetric about $\mathrm{Im}(\sigma_{13})=0$. For $\Phi=\pi/2$, only when the coupling field G_c becomes strong enough, for example $G_c=10\gamma$, can we obtain the probe gain without inversion around $\Delta_p=0$; otherwise the probe is always absorbed. While for $\Phi=3\pi/2$, when the coupling field is not very strong, we can obtain the probe gain in a much larger spectral range around $\Delta_p=0$. It is easy to find that, the maximal gain amplitude with $\Phi=0$ or $\Phi=\pi$ is very small compared to that with $\Phi=\pi/2$ or $\Phi=3\pi/2$. So, in the case of a strong microwave field, in order to obtain large enough probe gain, we should tune the relative phase into regions of $\Phi\cong\pi/2$ or $\Phi\cong3\pi/2$.

In order to investigate the response of the probe gain to the relative phase Φ in different cases, we plot the probe gain (or absorption) $\mathrm{Im}(\sigma_{13})$ at $\Delta_p=0$ *versus* Φ for different values of G_c and G_d in Figure 4. Obviously, the probe gain (or absorption) $\mathrm{Im}(\sigma_{13})$ is a periodical function of the relative phase Φ with the period of 2π, which is not limited to values of G_c and G_d. From Figure 4a, we can see that, even if in the case of $G_c=G_d=\gamma$ (both the microwave field and the coupling field are very weak), we can obtain the probe gain by tuning the relative phase Φ into proper regions. When the coupling field G_c becomes stronger, we can obtain much larger probe gain at $\Delta_p=0$, but we have to tune the relative phase Φ into regions of $2k\pi<\Phi<(2k+1)\pi$, where k is an integer. Note, the probe gain reaches a saturated value when G_c is near to 5γ. For $G_d=5\gamma$, as shown by Figure 4b, when the coupling field is not very strong, we can achieve the probe gain in the regions of $(2k-1)\pi<\Phi<2k\pi$. But when the coupling field becomes strong enough, for example $G_c=10\gamma$, in order to obtain the probe gain at $\Delta_p=0$, we should tune the relative phase into regions of $2k\pi<\Phi<(2k+1)\pi$. So, both amplitudes and phases of the applied fields are important factors that can remarkably affect gain behavior of the probe.

Now, we give out a qualitative description of the origin of the phase sensitivity of the probe gain or absorption. In the case of $\Omega_d=0$, the excited emission of electrons from level $|3\rangle$ to level $|1\rangle$ (or absorption from $|1\rangle$ to $|3\rangle$) only can take place in one way: $|3\rangle\rightarrow|1\rangle$ (or $|1\rangle\rightarrow|3\rangle$), and the emitted (or absorbed) photons have identical initial phases of ϕ_p, so the gain or absorption coefficient $\mathrm{Im}(\sigma_{13})$ does not depend on phases of the probe and coupling fields ϕ_p

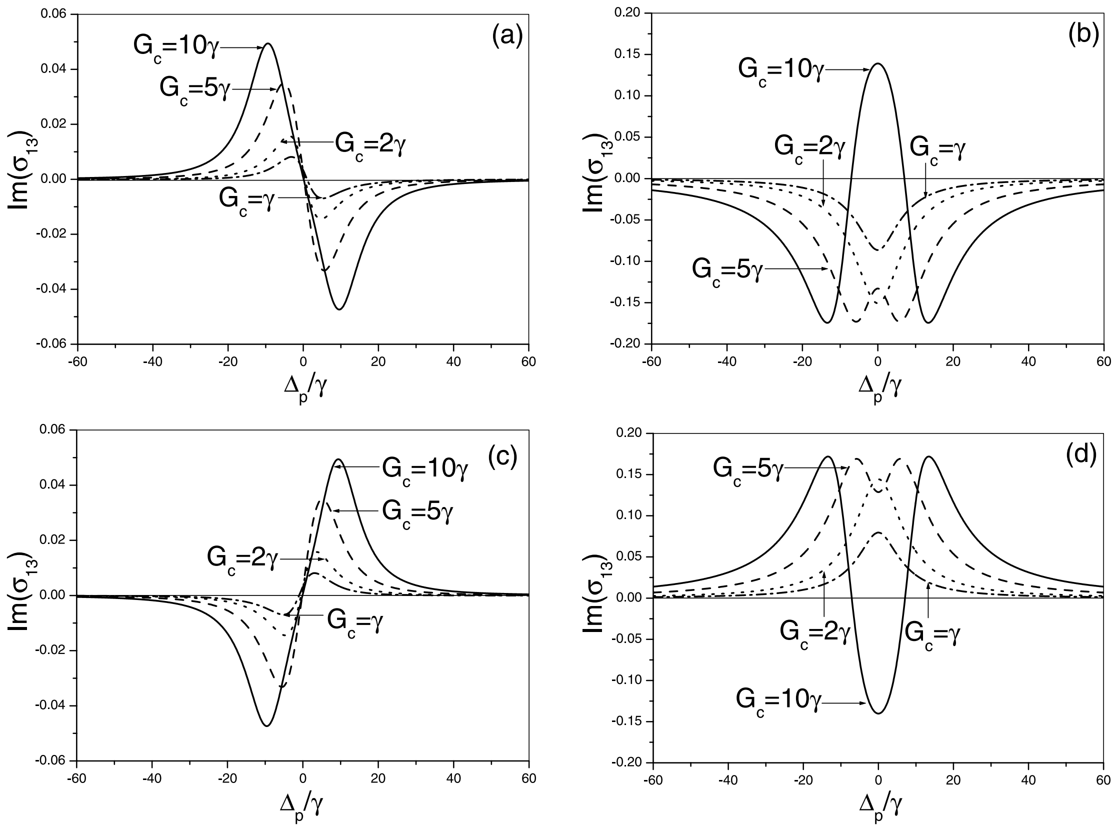

Fig. 3. The probe gain-absorption $\mathrm{Im}(\sigma_{13})$ against the probe detuning Δ_p/γ for (a) $\Phi = 0$; (b) $\Phi = \pi/2$; (c) $\Phi = \pi$; (d) $\Phi = 3\pi/2$ and different values of G_c. Other parameters are $\gamma_{31} = \gamma_{32} = \gamma$, $\Delta_\mathrm{c} = 0$, $G_\mathrm{p} = 0.1\gamma$, and $G_\mathrm{d} = 5\gamma$.

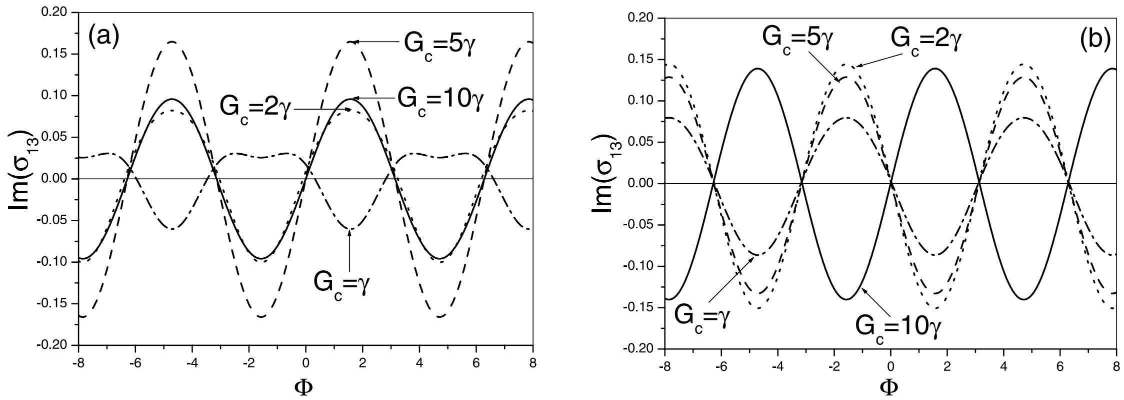

Fig. 4. The probe gain-absorption $\mathrm{Im}(\sigma_{13})$ at $\Delta_\mathrm{p} = 0$ against the relative phase Φ for (a) $G_\mathrm{d} = \gamma$; (b) $G_\mathrm{d} = 5\gamma$ and different values of G_c. Other parameters are the same as those in Figure 2.

and ϕ_c. But when the microwave field is included in this atomic system, as shown by Figure 5, there exist two different channels for the excited emission of electrons from level $|3\rangle$ to level $|1\rangle$ (or absorption from $|1\rangle$ to $|3\rangle$): the direct one $|3\rangle \to |1\rangle$ and the indirect one $|3\rangle \to |2\rangle \to |1\rangle$ (or $|1\rangle \to |3\rangle$ and $|1\rangle \to |2\rangle \to |3\rangle$). Photons absorbed or emitted through the direct channel have identical initial phases of ϕ_p, but those through the indirect channel have initial phases of $\phi_\mathrm{c} + \phi_\mathrm{d}$, for the indirect channel involves the absorption or emission of a microwave photon. These two absorption or emission channels interfere with one another and the total absorption or gain amplitude is the sum of two distinct components related to ϕ_p or $\phi_\mathrm{c} + \phi_\mathrm{d}$. Hence, the dependence of the probe gain $\mathrm{Im}(\sigma_{13})$ on phase difference $\Phi = \phi_\mathrm{c} + \phi_\mathrm{d} - \phi_\mathrm{p}$ is the consequence of interference between two competing transition channels.

4 Conclusions

In summary, we have shown that, in the atomic system considered above, we can achieve quite high inversionless

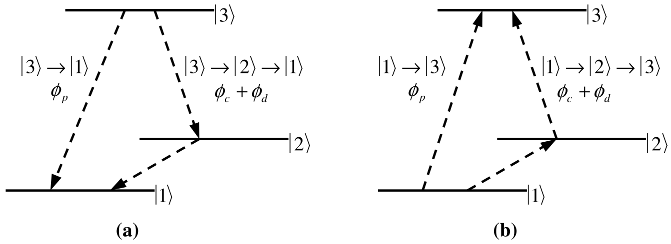

Fig. 5. Schematic representation of the first-order processes involved in (a) the excited emission of atoms in level $|3\rangle$; (b) the absorption of atoms in level $|1\rangle$.

gain on the probe transition, and the inversionless gain is related to the relative phase Φ of applied fields. The phase-sensitive effect of the probe gain arises from the quantum coherence between levels $|1\rangle$ and $|2\rangle$ induced by the applied microwave field, *i.e.*, the quantum interference between two transition channels $|1\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |2\rangle \leftrightarrow |3\rangle$. Of course, we also can control the gain behavior of the probe by changing strengths of the microwave field and the coupling field. Specifically, we show that the response of the inversionless gain at $\Delta_{\rm p} = 0$ to the relative phase Φ could be quite different or exactly opposite in different cases. That is to say, for some values of Φ, the probe gain at $\Delta_{\rm p} = 0$ can changes into absorption if we increase or decrease the amplitudes $G_{\rm c}$ or $G_{\rm d}$.

What we would like to emphasize here is that (a) the probe gain without inversion is achieved in the absence of incoherent pumping; (b) even if both the microwave field and the coupling field are not very strong, we still can obtain the phase-sensitive probe gain with quite high amplitude; (c) this scheme is very convenient and feasible in the experimental realization of phase control of the probe gain, for dipole moments $\mathbf{d}_{31}$ and $\mathbf{d}_{32}$ can be either parallel or orthogonal, ω_{21} can be much larger than γ_{31} and γ_{32}, and no incoherent pumping is needed.

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